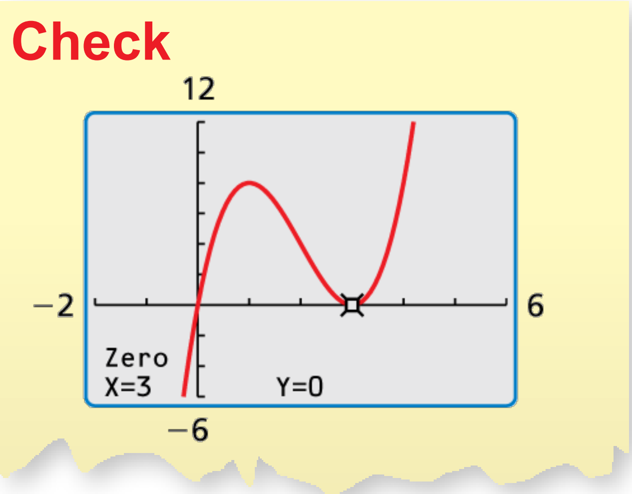
LESSON 4.5a

Solving Polynomial Equations

Today you will:

- Find solutions of polynomial equations and zeros of polynomial functions.
- Practice using English to describe math processes and equations

Solve $2x^3 - 12x^2 + 18x = 0$.



SOLUTION

$2x^3 - 12x^2 + 18x = 0$	Write the equation.
$2x(x^2 - 6x + 9) = 0$	Factor common monomial.
$2x(x - 3)^2 = 0$	Perfect Square Trinomial Pattern
$2x = 0$ or $(x - 3)^2 = 0$	Zero-Product Property
$x = 0$ or $x = 3$	Solve for x.

The solutions, or roots, are $x = 0$ and $x = 3$.

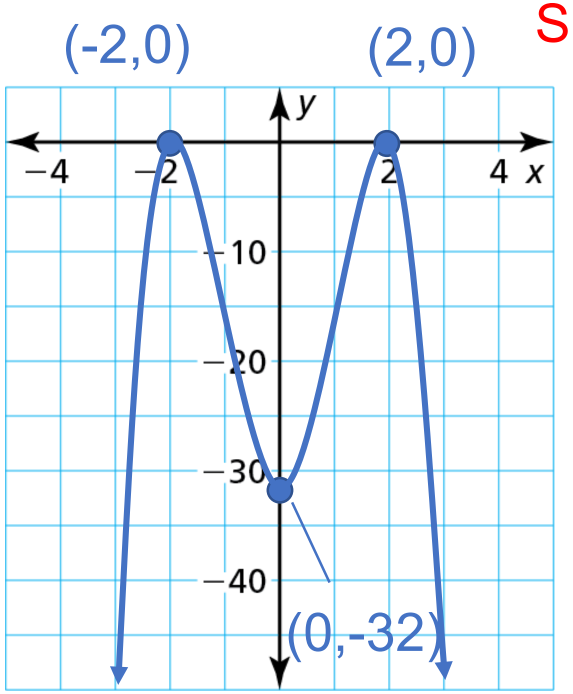
Note the factor $x - 3$ appears more than once.

Core Vocabulary: Repeated Solution (p. 180)

- When a factor $x - k$ of $f(x)$ is raised to an ***even*** power, the graph of f ***touches*** (but does not cross) the x-axis at $x = k$.
- When a factor $x - k$ of $f(x)$ is raised to an ***odd*** power, the graph of f ***crosses*** the x-axis at $x = k$.

Find the zeros of $f(x) = -2x^4 + 16x^2 - 32$. Then sketch a graph of the function.

SOLUTION



$0 = -2x^4 + 16x^2 - 32$	Set $f(x)$ equal to 0.
$0 = -2(x^4 - 8x^2 + 16)$	Factor out -2.
$0 = -2(x^2 - 4)(x^2 - 4)$	Factor trinomial in quadratic form.
$0 = -2(x + 2)(x - 2)(x + 2)(x - 2)$	Difference of Two Squares Pattern
$0 = -2(x + 2)^2(x - 2)^2$	Rewrite using exponents.

Because both factors $x + 2$ and $x - 2$ are raised to an even power, the graph of f touches the x-axis at the zeros $x = -2$ and $x = 2$.

By analyzing the original function:

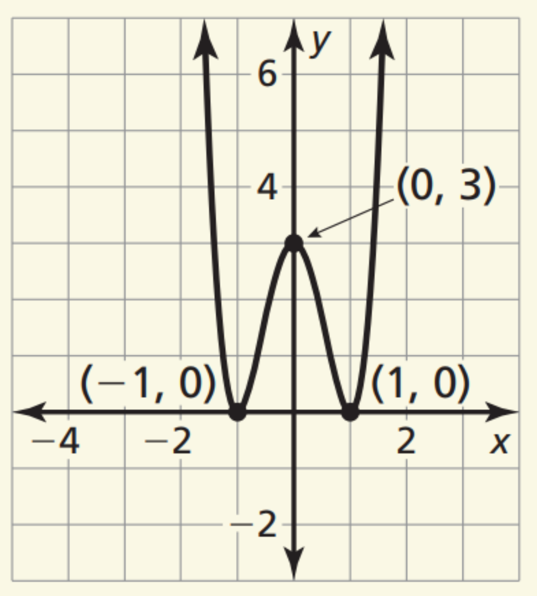
- You can determine that the y-intercept is -32.
- Because the degree is even is a W shape.
- Because the leading coefficient is negative it opens down: $f(x) \to \infty$ as $x \to -\infty$, $f(x) \to \infty$ as $x \to +\infty$.

Use these characteristics to sketch a graph of the function.

Find the zeros of $f(x) = 3x^4 - 6x^2 + 3$. Then sketch a graph of the function.

SOLUTION

$x = 1$ and $x = -1$



$0 = 3x^4 - 6x^2 + 3$	Set $f(x)$ equal to 0.
$0 = 3(x^4 - 2x^2 + 1)$	Factor out 3.
$0 = 3(x^2 - 1)(x^2 - 1)$	Factor trinomial in quadratic form.
$0 = 3(x + 1)(x - 1)(x + 1)(x - 1)$	Difference of Two Squares Pattern
$0 = 3(x + 1)^2(x - 1)^2$	Rewrite using exponents.

Because both factors $x + 1$ and $x - 1$ are raised to an even power, the graph of f touches the x-axis at the zeros $x = -1$ and $x = 1$.

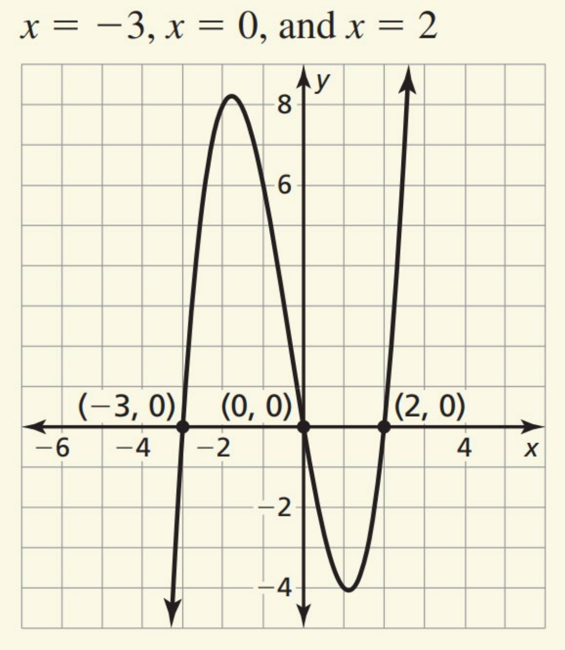
By analyzing the original function:

- You can determine that the y-intercept is 3.
- Because the degree is even is a W shape.
- Because the leading coefficient is positive it opens up: $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to +\infty$.

Use these characteristics to sketch a graph of the function.

Find the zeros of $f(x) = x^3 + x^2 - 6x$. Then sketch a graph of the function.

SOLUTION



$0 = x^3 + x^2 - 6x$ — Set $f(x)$ equal to 0.

$0 = x(x^2 + x - 6)$ — Factor out x.

$0 = x(x + 3)(x - 2)$ — Factor trinomial in quadratic form.

Because all factors x, $x + 3$ and $x - 2$ are raised to an odd power, the graph of f crosses the x-axis at the zeros $x = -3$, $x = 0$ and $x = 2$.

By analyzing the original function:

- You can determine that the y-intercept is 0.
- Because the degree is odd is a squiggle S shape.
- Because the leading coefficient is positive it ends pointing up: $f(x) \to \infty$ as $x \to \infty$, $f(x) \to \infty$ as $x \to -\infty$.

Use these characteristics to sketch a graph of the function.

Homework

Pg 194, #3-20